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LIMITS ON OBSERVATIONAL CAPABILITIES OF AEROSPACECRAFT

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SUMMARY

The principal limit on the observational capability of aerospacecraft is set by atmospheric turbulence. Most of the effect, however, is due to the distortions of the light path near the surface (up to 15 km), where the air density is high. Hence, the viewing accuracy of a satellite observer looking at the ground is generally much higher than for a ground observer viewing a satellite.

The ratios of these positional uncertainties have been estimated by assuming plausible or limiting relations for the instantaneous density gradients in the statistically fluctuating atmosphere. The resulting estimated uncertainty of viewing a point on the ground directly beneath an aerospacecraft need be no larger than 10 centimeters. This value is essentially independent of altitude above about 32 kilometers. Hence, very high flying aircraft would have about the same observational-capability limits as satellites. The minimum required telescope objective diameter, however, to achieve this 10-centimeter resolution must be increased with craft altitude up to 1.9 meters at an altitude of 320 kilometers.

INTRODUCTION

Limits on the observational capability of an observer or camera stationed on an aerospacecraft are largely determined by either the image brightness contrast of nearby areas or by the optical resolving power. The image brightness contrast is initially limited by contrast at the object with further reduction by intervening clouds, dust, smoke, and aerosols in the atmosphere, both because of opaqueness and the effects of scattered light (refs. 1 and 2). Consequently, the limits on observational capability set by image contrast varies widely from point to point and from time to time on the surface of the Earth.

The limits on the optical resolving power arise from distortions in the light ray paths due to atmospheric turbulence and local thermal gradients (refs. 3 and 4). The twinkling of stars viewed from the ground is one result. The fluctuations are largely

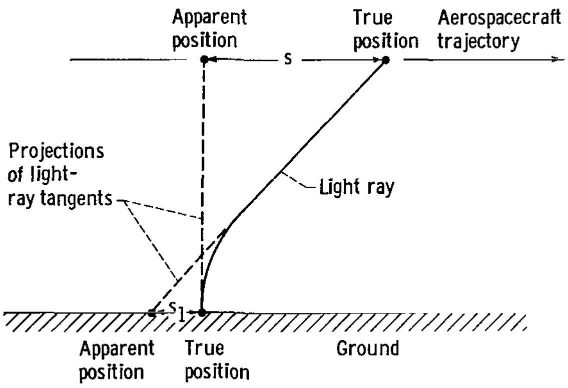

Figure 1. - Apparent image position for ground and aerospacecraft observer.

random in nature with a positional uncertainty of perhaps a maximum of 3 seconds of arc as observed from sea level. High-altitude observers might have an uncertainty of as low as 0.1 second of arc under rare favorable atmospheric conditions. Similar viewing uncertainties must persist in the observation of a satellite from the ground.

Three seconds of arc corresponds to an error of 4.7 meters in the position of a satellite located at a 320-kilometer altitude. The satellite observer, however, views the ground along the same instantaneous light path. Hence, at each instant as well as in a statistical sense, a relation should exist between the viewing accuracies of the ground and the satellite observers.

From figure 1, each observer appears to see his object along the projected tangent to the local light path. The light-path distortion, however, is large near the ground, where the air density is high, and is small near the aerospacecraft. Correspondingly, the position error in observing the aerospacecraft should be larger than the position error in observing the ground. Also, the error in observing the aerospacecraft increases with altitude, whereas the error in observing the ground is nearly independent of altitude if the aerospacecraft is essentially above the bulk of the atmosphere.

The study of this report was undertaken to estimate the uncertainty in atmospheric resolving power with which an aerospacecraft observer might view the ground directly below. The relative observational errors of the ground and the aerospacecraft observers are estimated by assuming plausible or limiting relations for the instantaneous density gradients in the statistically fluctuating atmosphere. Two cases are calculated. The first assumes a constant lateral density gradient with an exponential altitude decay. The second, also with exponential altitude decay, is a constant wavelength sinusoid. From these two cases, a plausible ratio is obtained for the observational errors of the reciprocally viewing ground and aerospacecraft observer. Utilization of the statistical angular positional uncertainty of a twinkling star then yields an estimate of the atmospheric-optical-resolving-power error for both the ground and the aerospacecraft observer. A brief discussion of other limits on the observational capability of an aerospacecraft is also included.

DERIVATIONS

A plane wave of light passing through a medium of variable index of refraction will

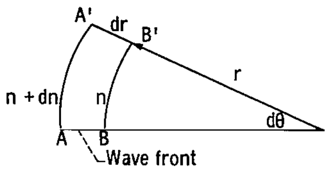

Figure 2. - Curvature of progressing light ray.

follow a curved path. The bending of the light ray is toward the portions of the medium having the larger index of refraction. The radius of curvature may be calculated from the component of the index of refraction gradient that is normal to the light path (fig. 2). (All symbols are defined in the appendix as well as in the text.) The time required for the wave front to move from A to A' must be the same as for the path B to B'. Hence,

$$\Delta t = \frac{(r + dr)(n + dn)d\theta}{c} = \frac{rn\ d\theta}{c} \tag{1}$$

where

Δt time of propagation

r radius of curvature of light path

n index of refraction

θ angular realinement of wave front

c speed of light in vacuum

When second-order terms are neglected, equation (1) yields, for air,

$$\frac{1}{r} = -\frac{1}{n}\frac{\partial n}{\partial r} \approx -\frac{\partial n}{\partial r} \tag{2}$$

The quantity $\partial n/\partial r$ is clearly the component of the gradient normal to the light-path direction.

The index of refraction is 1 in a vacuum. The quantity $n - 1$ is approximately proportional to the air density ρ. Hence,

$$n - 1 = k\frac{\rho}{\rho_{s\ell}} \tag{3}$$

The index at sea level is approximately 1.0003. Thus, if $\rho_{s\ell}$ is the sea-level air density, $k \approx 0.0003$.

The variation in atmospheric pressure p with altitude y may be approximated by the relation

$$p = p_{s\ell} e^{-y/y_o} \tag{4}$$

where the constant y_o may be adjusted to match the observed variation of pressure with altitude. For an isothermal atmosphere, the constant y_o is given as

$$y_o = \frac{RT_o}{wg} \tag{5}$$

where w, T_o, R, and g are, respectively, the average molecular weight of air, the average air temperature, the universal gas constant, and the acceleration of gravity. The pressure-density relation may be obtained from the universal gas law

$$\rho = \frac{pw}{RT} = \frac{p_{s\ell}w}{RT} e^{-y/y_o} \tag{6}$$

Hence, equation (3) becomes

$$n - 1 = \frac{kp_{s\ell}w}{\rho_{s\ell}RT} e^{-y/y_o} \tag{7}$$

The lateral variation of index required to bend a vertical light ray is a statistically fluctuating random quantity that depends on atmospheric turbulence, temperature gradients, and moisture content. No bending of a light ray can occur, however, where there is no air. Clearly, most of the bending of a light ray passing from the Earth to an aerospacecraft occurs in the low altitude regions, where the air density is high. Hence, any assumed relation for the lateral change of index of refraction with altitude due to all causes must contain a weighting factor that may be approximated by the exponential term e^{-y/y_o} of equation (7). Thus, even though the bending of a vertical light ray requires a lateral change in the index of refraction, the nominal strength of that change is limited by the predominating exponential decay with altitude.

The error in the position of a satellite near the zenith as observed from the ground will now be related to the lateral density gradient. Somewhat arbitrary relations for the lateral gradients will be assumed, and the corresponding errors in observing the satel-

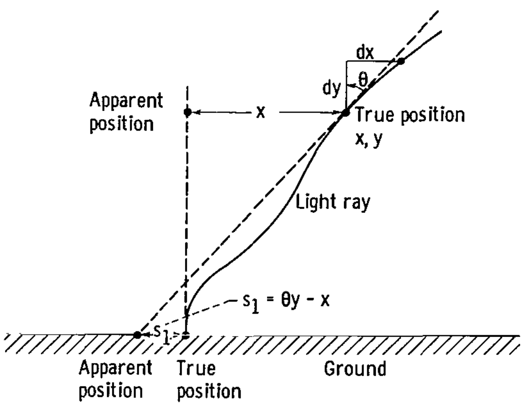

Figure 3. - Apparent and true positions of ground and aerospace observers.

lite and the ground along the same light path will then be compared. These gradients are chosen at a particular instant so that the time-dependent fluctuations are not involved.

Assume that the air density can be written as the product of two functions. One is the exponential decay factor of equation (7). The second may be a function of both the lateral coordinate x and the vertical coordinate y. The equation replacing equation (6) may then be written

$$\rho = \rho_{s\ell}[1 + f(x, y)]e^{-y/y_o} \tag{8}$$

The function $f(x, y)$ would probably be statistical in nature and might have a time average of zero. At the instant of this calculation, however, it generates a light-path radius of curvature given by substitution of equations (3) and (8) into equation (2) and replacement of ∂r with $-\partial x$:

$$\frac{1}{r} = k \frac{\partial f}{\partial x} e^{-y/y_o} \tag{9}$$

This equation recognizes that the curvature of the light path decays exponentially with altitude.

The apparent and the true positions of the ground and the aerospacecraft observers are shown on figure 3. The incremental error in position dx for each increment of altitude dy is

$$\frac{dx}{dy} = \tan \theta \approx \theta \tag{10}$$

where θ is the angular deviation from the otherwise vertical light path. From equation (9) and simple geometry,

$$d\theta = \frac{dy}{r \cos \theta} \approx \frac{dy}{r} = k \frac{\partial f}{\partial x} e^{-y/y_o} dy \tag{11}$$

Elimination of θ between equations (10) and (11) gives the approximate differential equation of the nearly vertical light path:

$$\frac{d^2x}{dy^2} = k\frac{\partial f}{\partial x} e^{-y/y_o} \tag{12}$$

Exact equations for rays in arbitrary directions are included in reference 5.

A special case will now be calculated for which f is the sum of two functions, one of x alone, the other of y alone. Linear approximation near $x = 0$ gives the following relation for f:

$$f = \left(\frac{\partial f}{\partial x}\right)_0 x + g(y) \tag{13}$$

This function will give a monotonically increasing value of θ with attitude. Integration of equation (11) yields

$$\theta = ky_o\left(\frac{\partial f}{\partial x}\right)_0\left(1 - e^{-y/y_o}\right) \tag{14}$$

The positional uncertainty s of the satellite as viewed from the ground is obtained by integration of equation (10):

$$s = ky_o^2\left(\frac{\partial f}{\partial x}\right)_0\left[\frac{y}{y_o} - \left(1 - e^{-y/y_o}\right)\right] \tag{15}$$

Equation (15) also gives the path of the light ray if s is replaced by x.

If a mean temperature of 256° K is assumed in equation (5), the quantity $y_o \approx 7.48$ kilometers. This value corresponds to a decrease in the exponential term e^{-y/y_o} and hence pressure by a factor of 2 for each 5.19-kilometer increase in altitude. The exponential term can thus be neglected in comparison to unity for altitudes above 30 kilometers. Equation (15) may, therefore, be approximated by

$$s = ky_o^2 \left(\frac{\partial f}{\partial x}\right)_0\left(\frac{y}{y_o} - 1\right) \tag{16}$$

The observational uncertainty associated with viewing an aerospacecraft from the ground thus increases essentially linearly with craft altitude above about 30 kilometers. For much higher altitudes, the ratio s/y is nearly equal to $ky_o(\partial f/\partial x)_0$. The value of k is about 0.0003 and $s/y \approx 1.46\times10^{-5}$ radian, corresponding to an angular swing of the light ray due to turbulence and thermal gradients of about 3 seconds of arc. The value of $(\partial f/\partial x)_0$ at the instant of maximum deviation is thus about 6.5×10^{-6} per meter for this example.

The assumption might be made that the value of $(\partial f/\partial x)_0$ results from a lateral thermal gradient. The density of the air and hence the index of refraction can thus change with x even though the pressure may not. From equations (2) and (3), the gas law, and equation (9),

$$\frac{1}{r} = \frac{\partial n}{\partial x} = \frac{k}{\rho_{s\ell}} \frac{\partial \rho}{\partial x} \approx - \frac{k}{T_o} \frac{\partial T}{\partial x} e^{-y/y_o} = k\left(\frac{\partial f}{\partial x}\right)_0 e^{-y/y_o}$$

If values of $T_o \approx 256^\circ$ K and $(\partial f/\partial x)_0 = 6.5\times10^{-6}$ are used, the angular deviation of the light ray can be caused by an average thermal gradient of order 1.7° K per kilometer, a value not at all unreasonable. Wind and turbulence could sweep such thermal gradients across the light-ray path to give the observed random directional fluctuations.

The observation may be made that the light path represented by equation (15) monotonically curves in the same direction. In the actual atmosphere, the curvature probably reverses with time and altitude. Reversals tend to cancel the positional uncertainties. Hence, the actual thermal gradients in the atmosphere might be larger than 1.7° K per kilometer, corresponding to the observed angular deviation of a twinkling star.

From figure 1 or 3, the uncertainty of the ground position as viewed from a satellite or aerospacecraft is

$$s_1 = \theta y - x = \theta y - s \tag{17}$$

Substitutions from equations (14) and (15) give

$$s_1 = k y_o^2 \left(\frac{\partial f}{\partial x}\right)_0 \left(1 - e^{-y/y_o} - \frac{y}{y_o} e^{-y/y_o}\right) \tag{18}$$

At 30 kilometers, the exponential terms together are less than one-tenth compared to unity and would be negligible above about 45 kilometers. Even at 30 kilometers, equation (18) can be approximated by

$$s_1 = k y_o^2 \left(\frac{\partial f}{\partial x}\right)_0 \tag{19}$$

Equation (19) shows that the uncertainty in viewing the ground from an aerospacecraft is essentially independent of altitude above 30 to 45 kilometers. The observational capability of a satellite as limited by the atmosphere is therefore almost as good as that of a very high flying aircraft.

From equations (16) and (19), the ratio of the observational errors of the ground and the satellite observer is

$$\frac{s}{s_1} = \frac{y}{y_o} - 1 \tag{20}$$

For a 320-kilometer satellite with an atmospheric mean temperature of 256^{o} K, this ratio is about 43. If a mean temperature is chosen to give the correct standard pressure at 30 kilometers via equation (4), the ratio s/s_1 is 47. Thus, the observational accuracy of a satellite observer is perhaps 45 times as great as that of the ground observer viewing the satellite along the same light path. The independence of this number from the strength of the initially assumed density gradient lends credence to the estimate.

An uncertainty of 3 seconds of arc in viewing a 320-kilometer satellite from sea level corresponds to a positional error of about 4.7 meters. Hence, the satellite observer can view the ground with an error of only about 10 centimeters. Thus, the observational capability of a satellite observer with the proper optical equipment can be superb. From equation (19) and the fact that any obscuring cloud cover would lie below 30 kilometers, the conclusion may be repeated that the observational capability of very high flying aircraft and satellites is essentially the same.

While the results of this simple example are interesting, the fact remains that they are based upon a statistically improbable model of the lateral density gradient. Some insight is therefore required to determine which portions of the atmosphere are most likely to produce the observational errors.

For a ground observer looking at an aerospacecraft, the maximum positional error increment dx for each increment in altitude dy along the light ray occurs when dx/dy or θ is a maximum. For the cited example, θ is maximum and constant in space, where there is no atmosphere. Perhaps a better criterion is then the condition for which $d\theta/dy$ is a maximum. From equation (11), the largest increments in θ occur when $(\partial f/\partial x)e^{-y/y_o}$ is maximized. For constant values of $\partial f/\partial x$, this maximum occurs at the Earth's surface.

On the other hand, the position error in seeing the ground from an aerospacecraft depends on both the angle the light ray makes to the vertical and the compensating light-ray deflection. From equations (17) and (11),

$$\frac{ds_1}{dy} = y\frac{d\theta}{dy} = yk\left(\frac{\partial f}{\partial x}\right)e^{-y/y_o} \tag{21}$$

For the cited example, $\partial f/\partial x$ is constant, and ds_1/dy is a maximum when $y = y_o \approx 7.5$ kilometers.

A somewhat more sophisticated example might be obtained by assuming temporarily that f varies sinusoidally in both the x- and y-directions:

$$f = f_m \sin\left(\frac{2\pi x}{a} + \frac{2\pi y}{b} + \varphi\right) \tag{22}$$

where f_m, a, b, and φ are constants at the instant of the calculation. Equation (12) for the light path then becomes

$$\frac{d\theta}{dy} = \frac{d^2x}{dy^2} = 2\pi f_m \frac{k}{a} \cos\left(\frac{2\pi x}{a} + \frac{2\pi}{b} y + \varphi\right)e^{-y/y_o} \tag{23}$$

Solutions to this equation will be considered for wavelengths a and b on the order of 30 meters or larger. From the previous example, the value of x, even at 30 kilometers, is less than 1 meter. Hence, the angular shift due to the term $2\pi x/a$ may generally be neglected. Equation (23) may be written thus

$$\frac{\partial^2 x}{dy^2} = k\left(\frac{\partial f}{\partial x}\right)_m \cos\left(\frac{2\pi y}{b} + \varphi\right)e^{-y/y_o} \tag{24}$$

where the constant $2\pi f_m/a$ is replaced by $(\partial f/\partial x)_m$. If $\theta = 0$ at $x = y = 0$, the first integration of equation (23) gives

$$\theta = \frac{dx}{dy} = k\left(\frac{\partial f}{\partial x}\right)_m \frac{\left[\frac{2\pi}{b}\sin\left(\frac{2\pi y}{b} + \varphi\right) - \frac{1}{y_o}\cos\left(\frac{2\pi y}{b} + \varphi\right)\right]e^{-y/y_o} - \frac{2\pi}{b}\sin\varphi + \frac{1}{y_o}\cos\varphi}{\left(\frac{2\pi}{b}\right)^2 + \left(\frac{1}{y_o}\right)^2} \tag{25}$$

A second integration gives the equation of the light path:

$$x = k\left(\frac{\partial f}{\partial x}\right)_m \left(\frac{\left\{ -\frac{4\pi}{by_o} \sin\left(\frac{2\pi y}{b} + \varphi\right) + \left[\left(\frac{1}{y_o}\right)^2 - \left(\frac{2\pi}{b}\right)^2\right]\left(\cos \frac{2\pi y}{b} + \varphi\right)\right\} e^{-y/y_o} + \frac{4\pi}{by_o} \sin \varphi - \left[\left(\frac{1}{y_o}\right)^2 - \left(\frac{2\pi}{b}\right)^2\right] \cos \varphi}{\left[\left(\frac{2\pi}{b}\right)^2 + \left(\frac{1}{y_o}\right)^2\right]^2} \right.$$

$$\left. + \frac{\left(+\frac{1}{y_o} \cos \varphi - \frac{2\pi}{b} \sin \varphi\right) y}{\left(\frac{2\pi}{b}\right)^2 + \left(\frac{1}{y_o}\right)^2} \right) \tag{26}$$

Above 30 to 45 kilometers, these quantities may be approximated by the equations

$$\theta = k\left(\frac{\partial f}{\partial x}\right)_m \frac{\frac{1}{y_o} \cos \varphi - \frac{2\pi}{b} \sin \varphi}{\left(\frac{2\pi}{b}\right)^2 + \left(\frac{1}{y_o}\right)^2} \tag{27}$$

$$x = s = k\left(\frac{\partial f}{\partial x}\right)_m \left\{ \frac{\frac{4\pi}{by_o} \sin \varphi - \left[\left(\frac{1}{y_o}\right)^2 - \left(\frac{2\pi}{b}\right)^2\right] \cos \varphi}{\left[\left(\frac{2\pi}{b}\right)^2 + \left(\frac{1}{y_o}\right)^2\right]^2} + \frac{\left(\frac{1}{y_o} \cos \varphi - \frac{2\pi}{b} \sin \varphi\right) y}{\left(\frac{2\pi}{b}\right)^2 + \left(\frac{1}{y_o}\right)^2} \right\} \tag{28}$$

From equation (17), s_1 is

$$s_1 = -k\left(\frac{\partial f}{\partial x}\right)_m \frac{\left\{ \frac{4\pi}{by_o} \sin \varphi - \left[\left(\frac{1}{y_o}\right)^2 - \left(\frac{2\pi}{b}\right)^2\right] \cos \varphi \right\}}{\left(\frac{2\pi}{b}\right)^2 + \left(\frac{1}{y_o}\right)^2} \tag{29}$$

Hence, if α is defined by equation (30),

$$\alpha = \frac{2\pi y_o}{b} \tag{30}$$

the value of s_1 is

$$s_1 = k y_o^2 \left(\frac{\partial f}{\partial x}\right)_m \frac{(1 - \alpha^2)\cos\varphi - 2\alpha\sin\varphi}{(1 + \alpha^2)^2} \tag{31}$$

Likewise, the ratio s/s_1 is

$$\frac{s}{s_1} = \frac{(1 + \alpha^2)(\cos\varphi - \alpha\sin\varphi)}{(1 - \alpha^2)\cos\varphi - 2\alpha\sin\varphi}\,\frac{y}{y_o} - 1 \tag{32}$$

If the wavelength b is very long, α approaches zero and equation (32) reduces to equation (20). Equation (31) for s_1 is then essentially equation (19) provided that $(\partial f/\partial x)_m \cos\varphi$ is replaced by $(\partial f/\partial x)_0$.

If the wavelength is short, α is large. For example, if b is 46 meters, α is about 1000. Hence, if $\alpha \tan\varphi$ is also much larger than unity, equations (31) and (32) may be approximated by

$$s_1 = -k\left(\frac{y_o}{\alpha}\right)^2 \left(\frac{\partial f}{\partial x}\right)_m \cos\varphi = -k\left(\frac{b}{2\pi}\right)^2 \left(\frac{\partial f}{\partial x}\right)_m \cos\varphi \tag{33}$$

and

$$\frac{s}{s_1} \approx (\alpha\tan\varphi)\,\frac{y}{y_o} - 1 = (2\pi\tan\varphi)\,\frac{y}{b} - 1 \tag{34}$$

The uncertainty of viewing the ground from a satellite at 320 kilometers is clearly less in this approximation than for the long wavelength case.

An intermediate case exists for which $\alpha \tan\varphi$ is approximately unity. The ratio s/s_1 for this situation could then feasibly be zero. From equation (27) the net angular shift of the light path is also nearly zero. A lateral shift occurs, however, in the light path given by equation (31) of magnitude

$$s_1 = ky_o^2\left(\frac{\partial f}{\partial x}\right)_m \frac{\cos \varphi}{1 + \alpha^2} \tag{35}$$

Inasmuch as $|\cos \varphi/(1 + \alpha^2)|$ cannot exceed unity, this error in position is again smaller than that of the long wavelength case.

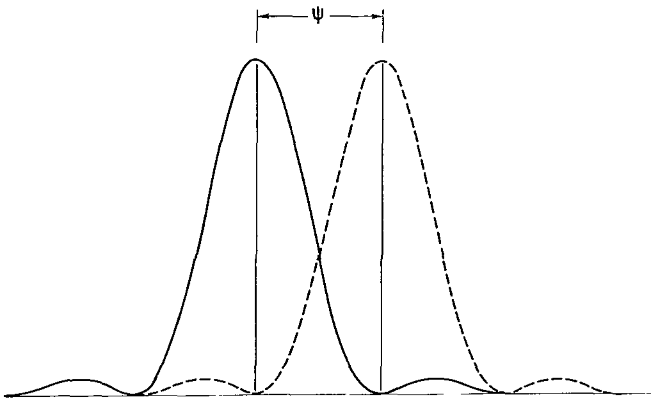

Figure 4. - Resolving power for circular aperture. The optical resolution angle $\psi = 1.22\,\lambda/d$, where λ is wavelength and d is diameter of aperture.

The two computed models for the lateral density gradients are obviously highly idealized. In the actual atmosphere, the turbulent processes are sufficiently random in nature and time dependent that even a damped sinusoidal density variation would be a poor approximation. Certainly the wavelength of the disturbance would vary from point to point. Also, the light-path distortion would not in general be limited to a single lateral direction as was assumed. The light ray might follow a randomly spiralled path up through the atmosphere with many variations in direction and curvature associated with atmospheric turbulence (ref. 6).

In the constant-density-gradient case the values of θ, s, and s_1 increased monotonically with altitude. For this sinusoidal case, θ and s_1 alternately increased and decreased with altitude, as may be seen from equations (23) and (21). The net result was a lower positional error in viewing a point on the ground from an aerospacecraft than was given by the constant-density-gradient case. By similar reasoning, the multidirectional three-dimensional distortions of the light path through the turbulent atmosphere would probably lead to estimates of position error no larger than those calculated from the constant-density-gradient case. Thus, the limits on the resolving power of the atmosphere as seen from an aerospacecraft are probably no greater than the 10 centimeters estimated herein.

OBSERVATIONAL LIMITS DUE TO OTHER CAUSES

Because of the wavelike nature of light, the circular aperture on an optical system generates a diffraction pattern for a point light source. The diffraction pattern for two point light sources in close proximity is shown on figure 4. Clearly, an observer would see only the combined intensities of both sources. These sources become hard to distinguish if the maximum of one is closer than the first minimum of the second. This criterion gives the well-known formula from the wave theory of light

$$\psi = 1.22 \frac{\lambda}{d} \tag{36}$$

where ψ is the minimum resolution angle, λ is the wavelength of the light, and d is the aperture diameter. This angle is also equal to the ratio of the minimum separation distance s_2 between two light sources that can be resolved and the distance y from the sources to the telescope objective. Hence,

$$\varphi = 1.22 \frac{\lambda}{d} = \frac{s_2}{y} \tag{37}$$

According to this formula, if $\lambda = 0.5$ micrometer, an observer on a satellite at a 320-kilometer altitude, using a telescope with a 30-centimeter-diameter objective, could distinguish two point light sources if they were separated by as little as 0.65 meter. This distance represents the fuzziness of the boundaries of real objects under observation. A larger telescope or a lower altitude will permit a smaller separation distance.

As judged by the previous calculations, atmospheric turbulence produces a random fuzziness of less than 10 centimeters in radius for a point object viewed from altitudes above 32 kilometers. The optical system needs no greater resolution. Hence, the required telescope objective for best resolutions as viewed from a 320-kilometer satellite requires a diameter of about 1.9 meters to resolve a 10-centimeter radius, or one-half this value if the diameter rather than the radius of the fuzziness due to the atmosphere is chosen. For the same viewing accuracy, an aerospacecraft flying at a 32-kilometer altitude would require a telescope diameter one-tenth as large. The limiting capability of both to see the ground is essentially the same. Both have the same cloud cover. Thus, the high flying aerospacecraft has the advantage over the satellite of being able to use smaller optical equipment for reconnaissance purposes. The aerospacecraft may also have greater maneuverability over the target.

The satellite must, of course, travel at a speed of about 7.6 kilometers per second to stay in orbit. To achieve the resolution discussed heretofore, either extremely short exposure times or motion compensation techniques would be required to follow an object on the ground. If television were employed, the line spacing in the instrument would have to be considered also.

The question might be raised as to whether sufficient illumination for observation exists. During the daytime, the visible light transmission coefficient through the entire atmosphere is about 85 percent at the zenith. Viewing the zenith through the entire atmosphere is about equivalent to looking at an object located horizontally about 8.5 kilometers away on the surface. Because of a favorable albedo, the brightness of the Earth from a satellite would be several times that of a full moon as viewed from the surface of the Earth.

The observational capability would also be useful at night. According to reference 7, the unaided human eye requires at least 2.5×10^{-9} erg per second of energy to detect a point light source. A 1-watt light bulb with 1-percent efficiency should therefore be observable from a 320-kilometer altitude with a 12-inch telescope. A photographic plate might require a 60-watt light bulb for a 1-minute exposure time, or perhaps less with the new fast films. Clouds, dust, and haze in the atmosphere would, of course, decrease the light available to the observer.

CONCLUSIONS

A study has been undertaken to estimate the limits on the observational capabilities of aerospacecraft. The following conclusions have been drawn:

1. The principal limit on the observational optical resolution capability of aerospacecraft is due to atmospheric turbulence. The most important light-path distortions originate near the surface (first 15 km), where the air density is high.

2. The positional uncertainty of a ground object as viewed from an aerospacecraft may be less than 10 centimeters. This uncertainty due to atmospheric turbulence is essentially independent of craft altitude above about 32 kilometers.

3. A satellite observer at an altitude of 320 kilometers can locate a point on the ground with an accuracy 45 times as great as that with which a ground observer looking along the same light path can locate a point on the satellite.

4. The diffraction pattern from the telescope objective does not fundamentally limit the viewing accuracy. The objective diameter must be large enough, however, that the diffraction-limited resolution equals or exceeds the value due to atmospheric turbulence.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 4, 1965.

APPENDIX - SYMBOLS

a	lateral wavelength (x-direction)
b	vertical wavelength (y-direction)
c	speed of light in vacuum
d	optical objective diameter
$f(x, y)$	dimensionless density function
$\left(\frac{\partial f}{\partial x}\right)_m$	$\frac{2\pi f_m}{a}$
$\left(\frac{\partial f}{\partial x}\right)_0$	value of $\frac{\partial f}{\partial x}$ near $x = 0$
f_m	amplitude of f in equation $f = f_m \sin\left(\frac{2\pi x}{a} + \frac{2\pi y}{b} + \varphi\right)$
g	acceleration due to gravity
k	proportionality constant in equation $n - 1 = k\rho/\rho_{\ell s}$
n	index of refraction
p	static pressure
$p_{s\ell}$	static pressure at sea level
R	universal gas constant
r	radius of curvature of light path
s	position uncertainty of aerospacecraft as viewed from sea level
s_1	position uncertainty of point at sea level as viewed from aerospacecraft (see fig. 1 or 3)
s_2	spacing of two point light sources, presumably equal to s_1
T	temperature
T_o	mean temperature chosen to approximate the variation of pressure with altitude
t	time
w	mean molecular weight of atmosphere
x	lateral direction coordinate
y	vertical direction coordinate
y_o	constant approximated by $\frac{RT_o}{wg}$
α	constant defined as $\frac{2\pi y_o}{b}$
θ	angular deviation of light ray from vertical
λ	wavelength of light
ρ	density of air
$\rho_{s\ell}$	density of air at sea level
φ	phase angle
ψ	optical resolution angle, $\frac{1.22\lambda}{d}$

REFERENCES

1. O'Keefe, John A.; Dunkelman, Lawrence; Soules, Stanley D.; Huch, William F.; and Lowman, Paul D., Jr.: Observations of Space Phenomena. Mercury Project Summary Including Results of the Fourth Manned Orbital Flight, May 15 and 16, 1963. NASA SP-45, 1963, pp. 327-347.

2. Middleton, W. E. K.: Vision Through the Atmosphere. Univ. of Toronto Press, 1952, p. 122 ff.

3. Tatarski, V. I. (R. A. Silverman, trans.): Wave Propagation in a Turbulent Medium. McGraw-Hill Book Co., Inc., 1961.

4. Reiger, S. H.: Starlight Scintillation and Atmospheric Turbulence. Astron. J., vol. 68, no. 6, Aug. 1963, pp. 395-406.

5. Luneburg, R. K.: Mathematical Theory of Optics. Univ. of Calif., Los Angeles, 1964, p. 27.

6. Chernov, L. A. (R. A. Silverman, trans.): Wave Propagation in a Random Medium. McGraw-Hill Book Co., Inc., 1960, Ch. II, pp. 12-34.

7. Russell, H. N.: Minimum Radiation Visually Perceptible. Astrophys. J., vol. 45, Jan. 1917, pp. 60-64.

NASA-Langley, 1965 E-2952

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